ent gate voltages. As long as $V_{GS}$ is low and does not reach pinchoff, the negative transconductance feature appears. At higher $V_{GS}$, the gate electric field becomes position dependent along the channel, which smears out the resonant coupling effect and its consequences. This leads to the crossing of $I_{DS}$-$V_{GS}$ curves. In the saturation region, the drain current $I_{DS}$ is mostly determined by the total number of electrons in the channel, resulting in the usual FET characteristics.

Fig. 2 indicates that this FET should function as an n-channel or p-channel-like device and also as a frequency multiplier, as long as the gate voltage is properly modulated. Fig. 4b shows one such example of a resistor-loaded circuit, which multiplies the frequency of the gate input by a factor of 3, as shown in Fig. 4a.

Fig. 4 Oscilloscope traces of input voltage $V_i = -0.3 V + 0.7 V \sin \omega t$, and output voltage of resistor-loaded circuit, where resistance is set at 1 k$\Omega$ and $V_G = 1 V$

a) Input voltage
b) Output voltage
Parameters are set to multiply the input frequency by 3.

In summary, we have demonstrated novel features in a double quantum well field-effect transistor and have shown its desirable device functionalities and also excellent compatibility with normal FETs in operation and fabrication process.

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References

NOVEL DIGITAL INTEGRATOR AND DIFFERENTIATOR

M. A. Al-Alaou1

Indexing terms: Digital circuits, Circuit design, Integrated circuits

A novel digital integrator and a novel digital differentiator are presented. Both the integrator and the differentiator are of first order and thus eminently suitable for real-time applications. Both have an almost linear phase. The integrator is obtained by interpolating two popular digital integration techniques, the rectangular and the trapezoidal rules. The resulting integrator outperforms both the rectangular and the trapezoidal integrators in range and accuracy. The new differentiator is obtained by taking the inverse of the transfer function of the integrator. The effective range of the differentiator is about 0.8 of the Nyquist frequency.

Basic concept: The basic concept came from observing that the ideal integrator response lies between the responses of the rectangular and trapezoidal rule [1]. It thus seems reasonable that interpolating the above two rules would result in a new integrator that could better approximate the ideal integrator.

New nonminimum phase digital integrator: It can be seen from Fig. 1 that the magnitude of the frequency response of the ideal integrator falls between the magnitudes of the rectangular and the trapezoidal integrators. This suggests approximating the ideal integrator as a weighted sum of the rectangular and trapezoidal integrators. At half the Nyquist frequency the ratio is about 1:3. Reflecting this observation to the transfer functions, the following relation between the transfer functions of the integrators is obtained. The subscript $N$ denotes the new integrator, $R$ denotes the rectangular integrator, and $T$ denotes the trapezoidal integrator:

$$H_N(z) = \frac{3}{4}H_R(z) + \frac{1}{4}H_T(z)$$

Substituting the corresponding transfer functions in eqn. 1, we obtain

$$H_N(z) = \frac{3}{4}\left(\frac{T}{z - 1} + \frac{1}{4}\frac{T(z + 1)}{2z - 1}\right)$$

Simplifying eqn. 2, we obtain the following transfer function of the new integrator:

$$H_N(z) = \frac{T}{6}\frac{(z + 1)}{(z - 1)}$$

Note that $T = 1$ in the frequency plots shown in Fig. 1, and

Fig. 1 Magnitude response of new integrator, and ideal, trapezoidal, and rectangular integrators, and Simpson integrator

<table>
<thead>
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<th>New</th>
<th>Ideal</th>
<th>Trapezoidal</th>
<th>Rectangular</th>
<th>Simpson</th>
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<td>Frequency</td>
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the Nyquist frequency is $\pi$. However in Figs. 2-6, the Nyquist frequency is normalised to 1. All the Figures in this Letter were obtained by using MATLAB* together with the Signal Processing ToolBox.*

Fig. 2 shows the percent relative magnitude error of the new integrator compared with the magnitude response of the ideal integrator. Fig. 3 shows the phase of the new integrator which is almost linear with a maximum phase error of 8.25° occurring at 0.55 of the Nyquist frequency. The ideal linear phase, shown as a dotted line, corresponds to an ideal integrator with half a sample of delay.

Fig. 4 shows the magnitude response of the new differentiator which approximates the ideal differentiator, shown as a dotted line, up to 0.78 of the Nyquist frequency with a 2% error. This compares favourably with the 5 point differentiator reported by Oppenheim and Schafer [4]. The percent relative magnitude error is the same as that of the new integrator shown in Fig. 2. Fig. 6 shows the phase response of the new differentiator which approximates the linear phase, shown as a dotted line, of the ideal differentiator with half a sample of delay. The maximum error is 8.25° occurring at 0.55 of the Nyquist frequency. This is better than the 10.5° occurring at 0.6 of the Nyquist frequency reported by Rabiner and Steiglitz [5] using higher order recursive differentiators. A midband frequency differentiator could be obtained by realising the pole at say $z = 0.99, \ldots$, instead of at $z = 1$. The resulting

New minimum phase digital integrator: To obtain a minimum phase digital integrator, reflect the zero at $z = -1$ to a zero at $z = -1/7$ and compensate for the magnitude appropriately by multiplying the resulting minimum phase transfer function by $r$ [2]. Thus the resulting transfer function of the minimum phase integrator is

$$H(z) = z^7 (z + 4)$$

$$8(z - 1)$$

$$8(z - 1)(z + 4)$$

(4)

The magnitude response is the same as that of the nonminimum phase integrator. The phase response of the minimum phase integrator is shown in Fig. 4.

New digital differentiator: In this Letter the proposed new differentiator is obtained by inverting the transfer function of the minimum phase integrator developed in the preceding Section [3].

The resulting transfer function of the new digital differentiator is

$$G(z) = \frac{8(z - 1)}{77(z + 4)}$$

$$^*$$ Trademark of The Mathworks Inc.

Fig. 5 shows the magnitude response of the new differentiator which approximates the ideal differentiator, shown as a dotted line, up to 0.78 of the Nyquist frequency within a 2% error. This compares favourably with the 5 point differentiator reported by Oppenheim and Schafer [4]. The percent relative magnitude error is the same as that of the new integrator shown in Fig. 2. Fig. 6 shows the phase response of the new differentiator which approximates the linear phase, shown as a dotted line, of the ideal differentiator with half a sample of delay. The maximum error is 8.25° occurring at 0.55 of the Nyquist frequency. This is better than the 10.5° occurring at 0.6 of the Nyquist frequency reported by Rabiner and Steiglitz [5] using higher order recursive differentiators. A midband frequency differentiator could be obtained by realising the pole at say $z = 0.99, \ldots$, instead of at $z = 1$. The resulting
Acknowledgment: A pleasure to thank T. Kailath for providing the atmosphere conducive to research by inviting me to spend the summer of 1991 at Stanford's Information Systems Lab. Here this work was initiated. I am grateful to B. McKee, M. Goldberg, D. Roy, L. Tong, N. Al-Dhahir, Guanghan Xu, H. Agbajum, A. Sayed and C. Schaper for their help during my stay at Stanford. This work was supported in part by the Research Board of The American University of Beirut.

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References

NOVEL POLARIMETRIC FIBRE DEVICE FOR INTERROGATING 'WHITE-LIGHT' INTERFEROMETERS

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Indexing terms: Optical sensors, Polarisation, Optical fibres

A novel, in-line, polarimetric fibre interferomer for 'white-light' interferometry is described. It consists of two equal lengths of polarization-maintaining fibre, spliced with their polarization axes orthogonal. The interferometer path difference is thermally tuned over the free-space equivalent of \( \pm 130 \mu \text{m} \) to allow matching to a remote sensor interferometer.

Introduction: 'White-light' interferometry (WLI) has, over the past few years, become an important fibre sensing technique [1-3]. It provides a means of identifying both the absolute optical interference fringe order and the residual phase angle in a remote, unbalanced interferometer. In a typical system, a broadband source is coupled through a remote sensing interferometer, such as a Fabry-Perot, with an optical path difference (OPD) \( d \). This acts as an optical filter which maps a periodic modulation on the initially smooth spectrum, the pitch of which characterizes the OPD of the sensor. In a time-domain description of the effect, light beams propagating via the two paths of the interferometer are only correlated if the light in the shorter path is subsequently delayed by a time \( 2d/\upsilon_p \), where \( \upsilon_p \) is the group velocity in the interferometer. If a value of \( d \) is chosen which is significantly larger than the correlation length of the broadband source, then, without such a compensating delay, small variations in \( d \) will not cause visible interference fringes in the integrated signal.

If, however, the optical signal is then passed through a second interferometer, whose path imbalance can be scanned, then high-visibility interference fringes will be detected when the magnitude of the path difference closely matches that in the remote interferometer. An independent measurement of the local delay thus allows determination of the magnitude of the unknown remote delay, free from ambiguities. The technique can give high accuracy, but it generally requires moving parts in the readout interferometer (e.g. the scanned Michelson [1]) or involves a relatively complex optical spectrometer arrangement (e.g. a dispersive element plus CCD detector).

We present a new method of constructing a thermally-scanned polarimetric interferometer, using two, nominally equal, lengths of polarisation-maintaining (PM) fibre, spliced together with their polarisation axes at 90°. The arrangement is essentially that of the compensated polarimetric interferometer which has been used for differential sensing [4]. If not mechanically strained, the OPD between the two polarisation modes in the polarimetric interferometer depends primarily on the temperature difference \( \Delta T = T_1 - T_2 \) between the two fibre lengths. By scanning the temperature difference, a controllable differential mode delay can be generated, for matching the OPD of the remote interferometer. The advantages of this technique over other methods are that the readout interferometer is a single length of fibre, without any moving parts, and the whole system may be spliced into an optically efficient and compact unit. In addition, operation is possible in wavelength regions where detector arrays are less readily available, and the single small-area detector possible with this technique offers lower noise than with self-scanned arrays.

Experiment: Fig. 1 depicts the thermally-scanned, in-line fibre WLI interrogation system. Light from a 1550 nm fibre-pigtailed ELED of 1/e² bandwidth ±42 nm, is coupled, via a directional fibre coupler into a Fabry-Perot interferometer, having a separation \( d \) to simulate a remote sensor. The power level at the sensor was 4μW. The Fabry-Perot cavity was formed between the end face of a single mode fibre and a reflective glass block mounted on a positioner. The rear surface of the block was rough-ground and painted to avoid spurious reflections. The low Fresnel reflectivity at each surface (<4%) meant that the Fabry-Perot effectively acted as a low finesse, two-beam (rather than multiple-beam) interferometer. The effective delay time of the sensor head was therefore 2d/\upsilon, where \( \upsilon \) is the velocity of light.

The receiving polarimetric interferometer was formed from two lengths (L₀ = 10097 mm, L₁ = 10073 mm) of PM fibre (York Ltd. HB1550) with beatlength 3.06 mm. In our case, \( L_0 \) and \( L_1 \) were first matched to within 24 ± 0.5 mm, giving a mismatch of only seven fringes when the fibres are of equal temperatures. Initial matching was carried out using a commercial York St6 chromatic dispersion instrument. Subsequent shortening of one fibre reduced the mismatch to less than one fringe. Equal optical power was launched into the fast and slow eigenaxes, i.e. there were equal intensity beams travelling in the fast and slow states. At the central splice, fast and slow axes are interchanged, so that for equal lengths and temperatures (\( L_0 = L_1 \), \( T_1 = T_2 \)) the differential delay is zero. Each length of PM fibre was contained within separate stain-