Researchers at the University of Queensland in Australia claimed on August 16, 2002 a world first - a flight-test of supersonic combustion-an air-breathing supersonic engine, tagged as a scramjet.

**Supersonic turbulent fuel-air mixing and evaporation**

An experimental jet broke a world speed record on Nov. 17, 2004, cruising at around 7000 mph (M~ 10) in a NACA test of cutting edge Scramjet engine technology.
Plan of presentation

• Objective
• The need to understand fuel-air mixing
• Governing equations
• Discretization procedure
• Solution algorithm
• Results
• Closing remarks
Objective

The objective of this work is to formulate, implement, and test following the Eulerian approach, an all speed control volume-based numerical procedure for predicting turbulent mixing and evaporation of droplets.
The need to understand fuel-air mixing

In supersonic combustors, the high flow velocities and short residence times require that fuel and air mix and burn quickly to avoid excessive long combustors. Therefore, understanding the nature of turbulent mixing and the need for means to augment the mixing process are essential in designing more efficient propulsive systems.
A ramjet has no moving parts and achieves compression of intake air by the forward speed of the air vehicle. Air entering the intake of a supersonic aircraft is slowed by aerodynamic diffusion created by the inlet and diffuser to velocities comparable to those in a turbojet augmenter. The expansion of hot gases after fuel injection and combustion accelerates the exhaust air to a velocity higher than that at the inlet and creates positive push.
The Scramjet Engine

Scramjet is an acronym for Supersonic Combustion Ramjet. The scramjet differs from the ramjet in that combustion takes place at supersonic air velocities through the engine. It is mechanically simple, but vastly more complex aerodynamically than a jet engine.
Governing equations

- Gas Balance Equations
- Droplet Balance Equations
- Turbulence Closure
Governing equations (cont.)

• Gas Balance Equations

\[ \frac{\partial}{\partial t} (\rho_g v_g ) + \nabla \cdot (\rho_g \rho_g v_g v_g ) = \nabla \left( \frac{\mu_{t,g}}{S_{t,g}} \nabla \alpha_g \right) + I_{c,g} \]

\[ \frac{\partial}{\partial t} (\alpha_g \rho_g v_g ) + \nabla \cdot (\alpha_g \rho_g v_g v_g ) = -\alpha \nabla p + \nabla \bar{\tau}_g + F_g^B + F_g^D + I_{M,g} \]

\[ \frac{\partial}{\partial t} (\alpha_g \rho_g h_g ) + \nabla \cdot (\alpha_g \rho_g v_g h_g ) = -\nabla \cdot \dot{q}_g + \nabla \left( \frac{\mu_{t,g}}{Pr_{t,g}} \nabla h_g \right) + I_{E,g} \]

\[ \frac{\partial}{\partial t} (\alpha_g \rho_g Y_{vap,g} ) + \nabla \cdot (\alpha_g \rho_g v_g Y_{vap,g} ) = \nabla \left( \alpha_g \Gamma_{vap,g} \nabla Y_{vap,g} \right) + M_{vap,g} \left( 1 - Y_{vap,g} \right) \]
**Governing equations (cont.)**

- **Droplet Balance Equations**

\[
\frac{\partial}{\partial t} (\alpha_d \rho_d) + \nabla \cdot (\alpha_d \rho_d \mathbf{v}_d) = \nabla \cdot \left( \frac{\mu_{t,d}}{Sc_{t,d}} \nabla \alpha_d \right) + I_{C,d}
\]

\[
\frac{\partial}{\partial t} (\alpha_d \rho_d \mathbf{v}_d) + \nabla \cdot (\alpha_d \rho_d \mathbf{v}_d \mathbf{v}_d) = -\alpha \nabla p + \nabla \mathbf{\tau}_d + \mathbf{F}_d^B + \mathbf{F}_d^D + I_{M,d}
\]

\[
\frac{\partial}{\partial t} (\alpha_d \rho_d h_d) + \nabla \cdot (\alpha_d \rho_d \mathbf{v}_d h_d) = \nabla \cdot \left( \frac{\mu_{t,d}}{Pr_{t,d}} \nabla h_g \right) + I_{E,d}
\]

\[
\frac{\partial}{\partial t} (\alpha_d \rho_d D) + \nabla \cdot (\alpha_d \mathbf{v}_d \rho_d D) = \nabla \cdot \left( \alpha_d \frac{\mu_{t,d}}{Pr_{t,d}} \nabla D_d \right) - \frac{8\alpha_d}{\pi D^2} \dot{m}_{vap}^*
\]
Governing equations (cont.)

• Turbulence Closure

• Gas Phase

\[
\frac{\partial}{\partial t} \left( \alpha_g \rho_g k \right) + \nabla \cdot \left( \alpha_g \rho_g \mathbf{v}_g k \right) = \nabla \cdot \left( \alpha_g \frac{\mu_{\text{eff},g}}{\sigma_k} \nabla k \right) + \alpha_g \left( P_k - \rho_g \varepsilon \right) + S_{k,d}
\]

\[
\frac{\partial}{\partial t} \left( \alpha_g \rho_g \varepsilon \right) + \nabla \cdot \left( \alpha_g \rho_g \mathbf{v}_g \varepsilon \right) = \nabla \cdot \left( \alpha_g \frac{\mu_{\text{eff},g}}{\sigma_{T,\varepsilon}} \nabla \varepsilon \right) + \alpha_g \left( \frac{C_{\varepsilon_1}}{k} P_k - C_{\varepsilon_2} \rho_g \frac{\varepsilon^2}{k} \right) + S_{\varepsilon,d}
\]

\[\mu_{\text{turb},g} = C_\mu \rho_g \frac{k_g^2}{\varepsilon_g} \]
Governing equations (cont.)

• Turbulence Closure

• Droplet Phase

To simulate the effect of turbulent spray dispersion, the gas velocity is randomly sampled along the droplet trajectories and the characteristic quantities of the turbulence structure are determined from mean gas flow properties according to:

\[ \mu_{\text{turb,d}} = \mu_{\text{turb,g}} \frac{\rho_d}{\rho_g} \frac{k_d}{k_g} \]

\[ \frac{k_d}{k_g} = \frac{1}{1 + \omega^2 \tau^2} \]

\[ L_x = \left( c_{\mu} \right)^{3/4} \left( \frac{k_g}{\varepsilon_g} \right)^{3/2} \]

\[ \omega = \frac{1}{\tau} \left( \frac{\sqrt[3]{2} k_g}{L_x} \right)^{1/4} \]

\[ \tau = \frac{1}{18 \rho_g \nu} \frac{D^2}{1 + 0.133 Re_d^{0.687}} \]

\[ L_x \text{ and } \tau \text{ being the length scale and dissipation time scales of the idealized eddies.} \]
Discretization procedure

General Conservation Equation

\[
\frac{\partial}{\partial t} \left( r^{(k)} \rho^{(k)} \phi^{(k)} \right) + \nabla \cdot \left( r^{(k)} \rho^{(k)} u^{(k)} \phi^{(k)} \right) = \nabla \cdot \left( r^{(k)} \Gamma^{(k)} \nabla \phi^{(k)} \right) + r^{(k)} Q^{(k)}
\]

Algebraic Forms

\[
\alpha_P^\phi \phi_P = \sum_{NB(P)} \alpha_{NB}^\phi \phi_{NB} + b_P^\phi
\]

\[
\phi^{(k)} = H_P \left[ \phi^{(k)} \right] -
\]

\[
u_P^{(k)} = H_P P \left[ u^{(k)} \right] - r^{(k)} D_P^{(k)} \nabla_P (P)
\]
Solution algorithm

➢ The Pressure correction equation:

\[
\sum_k \left\{ \frac{\left( r_p^{(k)} \rho_p^{(k)} \right) - \left( r_p^{(k)} \rho_p^{(k)} \right)^{old}}{\delta t} \right\} \Omega + \Delta P \left( r_p^{(k)} \rho_p^{(k)} \mathbf{u}^{(k)} \cdot \mathbf{S} \right) = 0
\]

\[
\Delta P [\Theta] = \sum_{f=nb(P)} \Theta_f
\]

\[
P = P^* + P', \quad \mathbf{u}^{(k)} = \mathbf{u}^{(k)}^* + \mathbf{u}^{(k)'}, \quad \mathbf{n}^{(k)} = \mathbf{n}^{(k)} + \mathbf{n}^{(k)'}
\]

\[
\sum_k \left\{ \frac{1}{\Delta P} r_p^{(k)} \rho_p^{(k)} \left( r_p^{(k)} \mathbf{D}^{(k)} \nabla P' \right) \mathbf{S} \right\} \Omega + \Delta P \left[ r_p^{(k)} \rho_p^{(k)} \mathbf{U}^{(k)'} \right] = 0
\]

\[
a^\phi_P P' = \sum_{NB(P)} a^\phi_{NB} P_{NB} + b^\phi_P
\]

\[
\mathbf{u}_P^{(k)} = \mathbf{u}_P^{(k)} - \alpha^{(k)} \mathbf{D}_P^{(k)} \nabla P', P^* = P^* + P', \quad \rho = \rho^* + C_\rho P'
\]
The sequence of events is as follows:

- Solve the fluidic momentum equations for velocities.
- Solve the pressure correction equation based on global mass conservation.
- Correct velocities, densities, and pressure.
- Solve the fluidic mass conservation equations for volume fractions.
- Solve the fluidic scalar equations (k, e, T, Y, D, etc…).
- Return to the first step and repeat until convergence.
Results and discussion

- Evaporation and Mixing of water droplets injected parallel to a stream of air:
  - Subsonic turbulent evaporation and mixing
  - Supersonic turbulent evaporation and mixing

![Diagram showing dimensions](image)

- $L = 1$ m
- $W = 0.25$ m
- $d = 0.002$ m
Results and discussion (cont.)

➢ Turbulent evaporation and mixing of water droplets injected at right angle in a supersonic stream of air

\[ L = 1 \text{ m} \]
\[ W = 0.25 \text{ m} \]
\[ d = 0.002 \text{ m} \]
Subsonic turbulent evaporation and mixing

\[ M_{\text{inlet}} = 0.2 \quad T_g = 900 \text{ K} \quad T_d = 300 \text{ K} \]

Fig. 1  (a) Velocity profile, (b) droplet volume fraction, (c) gas temperature, (d) water vapor fraction, and (e) pressure field.

Laminar

Turbulent
Subsonic turbulent evaporation and mixing

Fig. 2  Variation of droplet temperature and diameter along the center line of the domain.
Supersonic turbulent evaporation and mixing

\[ M_{\text{inlet}} = 2, \quad T_g = 900 \text{ K} \quad T_d = 300 \text{ K} \]

Fig. 3 (a) Velocity profile, (b) droplet volume fraction, (c) gas temperature, (d) water vapor fraction, and (e) pressure field.
Supersonic turbulent evaporation and mixing

Fig. 4 Variation of droplet temperature and diameter along the center line of the domain.
Supersonic turbulent evaporation and mixing

Fig. 5 (a) The percentage, by mass, of the injected liquid droplets that evaporates at different inlet gas temperatures. 
(b) The percentage, by mass, of the injected liquid droplets that evaporates at different inlet droplet temperatures.
Supersonic turbulent evaporation and mixing

Fig. 6 The percentage, by mass, of the injected liquid droplets that evaporates for different domain lengths.
Supersonic turbulent evaporation and mixing with injection at right angle : case 1

\[ M_{\text{inlet}} = 2, \quad T_g = 900 \text{ K} \quad T_d = 300 \text{ K} \]

Fig. 7 (a) Velocity profile, (b) droplet volume fraction, (c) gas temperature, (d) water vapor fraction, and (e) pressure field.
Supersonic turbulent evaporation and mixing with injection at right angle: case 1

\[ M_{\text{inlet}} = 2, \quad T_g = 900 \, \text{K}, \quad T_d = 300 \, \text{K} \]

Fig. 8 Variation of droplet temperature and diameter along the center line of the domain.
Supersonic turbulent evaporation and mixing with injection at right angle: case 2

\[ M_{\text{inlet}} = 2. \quad T_g = 900 \text{ K} \quad T_d = 300 \text{ K} \]

Fig. 9 (a) Velocity profile, (b) droplet volume fraction, (c) gas temperature, (d) water vapor fraction, and (e) pressure field.
Closing remarks

- An Eulerian model for the prediction of variable size droplet evaporation and mixing at all speeds was presented.
- The model was tested by solving evaporation and mixing problems in the various Mach number regimes.
- Results are promising but still need validation against published data.
- Even though water was used for the droplets, this is not a model restriction and any other type of fuel can equally be used.